1. Solve Exercise 7 from Sheet 1.
2. Solve Exercise 8 from Sheet 2.
3. Show that the Lax--Wendroff method is consistent with order two in both time and space and that it is an approximation up to terms of order $\mathcal{O}(\tau^2 h^2)$ for the dispersive PDE

$$u_t + u_x = \frac{vh^2(\alpha^2 - 1)}{6} u_{xxx}, \qquad \alpha = \frac{v\tau}{h}.$$

4. Carry out a von Neumann stability analysis for the Lax--Wendroff method.
5. Solve the linear advection problem $u_t + u_x = 0, \quad 0 \leq x \leq 1, \quad 0 \leq t \leq 0.5$ on a computer with the upwind scheme

$$u_j^{n+1} = u_j^n - \tfrac{\tau}{h}\big(u_j^n - u_{j-1}^n\big)$$

and the Lax--Wendroff scheme

$$u_j^{n+1} = u_j^n - \tfrac{\tau}{2h}\big(u_{j+1}^n - u_{j-1}^n\big) + \tfrac{\tau^2}{2h^2}\big(u_{j+1}^n - 2u_j^n + u_{j-1}^n\big),$$

respectively. Try different values of h and τ and

(a) choose periodic boundary conditions (i.e.~$u(t,0) = u(t,1)$) and a smooth initial value, e.g.~$u_0(x) = \sin(2\pi x)$;

(b) choose an inflow condition at $x = 0$ (i.e.~$u(t,0)$ given) and a smooth initial value;

(c) plot the solutions and the errors.

6. Apply the method of characteristics to

$$\partial_t u(t,x) + v\partial_x u(t,x) = c(x)$$

Determine an exact solution for $c(x) = \mathrm{e}^{-x^2}$.